

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/05
Paper 5 Mechanics 2 (M2)
October/November 2008
1 hour 15 minutes
Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~m} \mathrm{~s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 One end of a light elastic rope of natural length 2.5 m and modulus of elasticity 80 N is attached to a fixed point $A$. A stone $S$ of mass 8 kg is attached to the other end of the rope. $S$ is held at a point 6 m vertically below $A$ and then released. Find the initial acceleration of $S$.

2


A uniform solid cylinder has height 24 cm and radius $r \mathrm{~cm}$. A uniform solid cone has base radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$. The cylinder and the cone are both placed with their axes vertical on a rough horizontal plane (see diagram, which shows cross-sections of the solids). The plane is slowly tilted and both solids remain in equilibrium until the angle of inclination of the plane reaches $\alpha^{\circ}$, when both solids topple simultaneously.
(i) Find the value of $h$.
(ii) Given that $r=10$, find the value of $\alpha$.

3 A particle $P$ of mass 0.5 kg moves along the $x$-axis on a horizontal surface. When the displacement of $P$ from the origin $O$ is $x \mathrm{~m}$ the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ in the positive $x$-direction. Two horizontal forces act on $P$; one force has magnitude $\left(1+0.3 x^{2}\right) \mathrm{N}$ and acts in the positive $x$-direction, and the other force has magnitude $8 \mathrm{e}^{-x} \mathrm{~N}$ and acts in the negative $x$-direction.
(i) Show that $v \frac{\mathrm{~d} v}{\mathrm{~d} x}=2+0.6 x^{2}-16 \mathrm{e}^{-x}$.
(ii) The velocity of $P$ as it passes through $O$ is $6 \mathrm{~m} \mathrm{~s}^{-1}$. Find the velocity of $P$ when $x=3$.


Fig. 1

A small sphere $A$ of mass 0.15 kg is moving inside a fixed smooth hollow cylinder whose axis is vertical. A moves with constant speed $1.2 \mathrm{~m} \mathrm{~s}^{-1}$ in a horizontal circle of radius 0.35 m , and is continuously in contact with both the plane base and the curved surface of the cylinder. Fig. 1 shows a vertical cross-section of the cylinder through its axis. Find the magnitude of the force exerted on $A$ by
(a) the base of the cylinder,
(b) the curved surface of the cylinder.
(ii)


Fig. 2
Sphere $A$ is now attached to one end of a light inextensible string. The string passes through a small smooth hole in the middle of the base of the cylinder. Another small sphere $B$, of mass 0.25 kg , is attached to the other end of the string. $B$ hangs in equilibrium below the hole while $A$ is moving in a horizontal circle of radius 0.2 m (see Fig. 2). Find the angular speed of $A$.

$A B C D$ is a central cross-section of a uniform rectangular block of mass 35 kg . The lengths of $A B$ and $B C$ are 1.2 m and 0.8 m respectively. The block is held in equilibrium by a rope, one end of which is attached to the point $E$ of a rough horizontal floor. The other end of the rope is attached to the block at $A$. The rope is in the same vertical plane as $A B C D$, and $E A B$ is a straight line making an angle of $20^{\circ}$ with the horizontal (see diagram).
(i) Show that the tension in the rope is 187 N , correct to the nearest whole number.
(ii) The block is on the point of slipping. Find the coefficient of friction between the block and the floor.

6 A light elastic string has natural length 4 m and modulus of elasticity 2 N . One end of the string is attached to a fixed point $O$ of a smooth plane which is inclined at $30^{\circ}$ to the horizontal. The other end of the string is attached to a particle $P$ of mass 0.1 kg . $P$ is held at rest at $O$ and then released. The speed of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$ when the extension of the string is $x \mathrm{~m}$.
(i) Show that $v^{2}=45-5(x-1)^{2}$.

Hence find
(ii) the distance of $P$ from $O$ when $P$ is at its lowest point,
(iii) the maximum speed of $P$.

7 A particle $P$ is projected from a point $O$ on horizontal ground with speed $V \mathrm{~m} \mathrm{~s}^{-1}$ and direction $60^{\circ}$ upwards from the horizontal. At time $t \mathrm{~s}$ later the horizontal and vertical displacements of $P$ from $O$ are $x \mathrm{~m}$ and $y \mathrm{~m}$ respectively.
(i) Write down expressions for $x$ and $y$ in terms of $V$ and $t$ and hence show that the equation of the trajectory of $P$ is

$$
\begin{equation*}
y=(\sqrt{ } 3) x-\frac{20 x^{2}}{V^{2}} . \tag{5}
\end{equation*}
$$

$P$ passes through the point $A$ at which $x=70$ and $y=10$. Find
(ii) the value of $V$,
(iii) the direction of motion of $P$ at the instant it passes through $A$.

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